Appendix P
Socket Finite Element Analysis

1.0 Introduction

We performed a series of finite element (FE) analyses to simulate the behavior of zinc-filled spelter sockets under load and determine how this behavior varies with some of the socket's properties. The FE analyses are complementary to the other approaches taken to analyze the failures of the telescope's sockets, which include a laboratory study (Appendix M), and full-scale load test (Appendix N), and the development of a mathematical model for the strength of sockets (Appendix O).

2.0 Modeling Assumptions

The FE models are built in Abaqus, which is a general-purpose FE analysis program. The general modeling assumptions are presented in this section and apply to all of the models and results presented in this appendix, unless noted otherwise.

2.1 Baseline Model Geometry

The FE models are derived from a baseline model representing the ground-end socket of an auxiliary backstay. One of these sockets experienced the maximum cable slip (B12W_G), and its overall geometry is similar to the sockets where the first two cable failures occurred (M4N_T and M4-4_T).

The wire broom is assumed uniform in the baseline model. In the uniform broom, the wire ends are evenly spaced in the radial and tangential directions. The uniform broom has a 60-degree axisymmetry, and therefore a single representative 60-degree wedge is modeled (Figure 1).

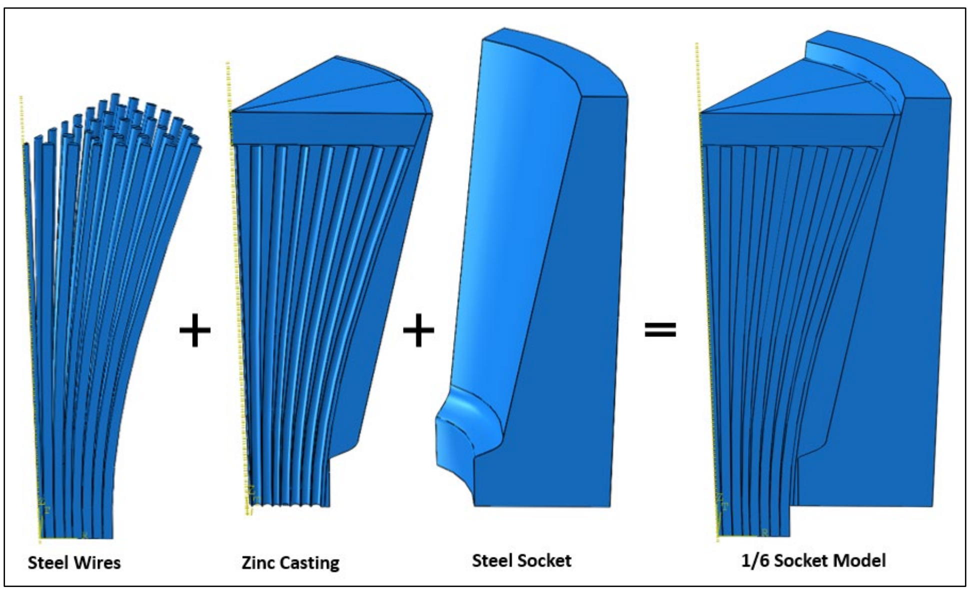

Figure 1: Finite element model of cable socket.

2.2 Material Properties

The steel wires and zinc casting are assigned nonlinear material properties to capture plastic deformation and load redistribution that may occur inside the socket (Table 1, Figure 2). The steel of the wires is modeled as an elastoplastic material with a yield stress of 175 ksi, which is the average yield stress measured on wires cut from the telescope's cables (Appendix L). The zinc is also modeled as an elastoplastic material, with a stress-strain curve based on the results of a test performed by WJE[1] on the zinc of socket M4N_T (first cable failure). In addition, the zinc is assigned a creep behavior to model zinc flow. The creep model is a power law where the creep rate is proportional to the fourth power of the stress. The steel of the socket is modeled as an elastic material.

The zinc material is allowed to slip on the socket's surface, with a friction coefficient of 0.6. The zinc can also separate from the socket's surface, as was observed at the back of some of the telescope's sockets. The wires and the zinc are tied, and therefore the wires cannot slip with respect to the zinc. Wire slip was observed in the zinc of some of the telescope's sockets, but its impact on the socket behavior and strength is not the focus of the FE analyses.

Table 1: FE model material properties.

Material parameter	Wire Steel	Zinc
Young's modulus, E [ksi]	29,000	12,490
Yield strength, F_y [ksi]	175	12.0
Poisson's ratio	0.3	0.245

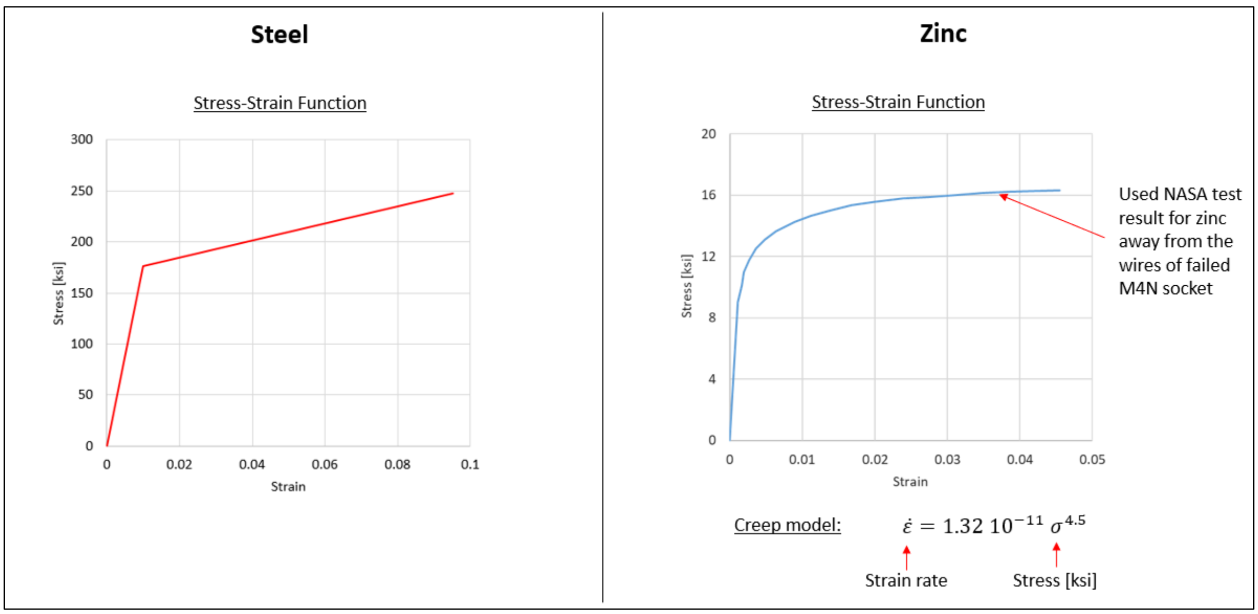

Figure 2: FE model material properties.

2.3 Mesh

The models are meshed (Figure 3) using a combination of linear hexahedral and tetrahedron elements (C3D8R and C3D4). The tetrahedron elements are used in the zinc material close to the wires to facilitate the meshing process. After performing a mesh sensitivity study, the maximum element size was selected at 0.2 inch.

[1] Wiss, Janney, Elstner Associates, Inc (WJE). *Auxiliary Main Cable Socket Failure Investigation*. June 21, 2021. Draft report provided by WJE.

2.4 Loading and Boundary Conditions

In the FE model, the socket is loaded by pulling on the back of the socket while fixing the cable end at the front of the socket (Figure 4). The socket is loaded to half of the cable's Minimum Breaking Strength, which is a stress level similar to what the telescope's sockets experienced. When analyzing the long-term behavior of a socket, the load is held for 25 years, which corresponds to the time between the installation of the auxiliary cables (started in 1995) and the telescope's collapse (2020). Additional boundary conditions are applied to the socket, zinc, and wires on the two sides of the 60-degree wedge to achieve axisymmetric behavior.

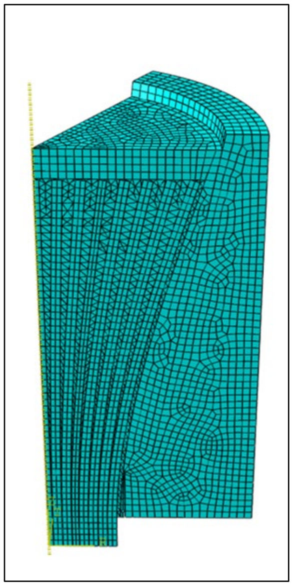

Figure 3: Finite element mesh.

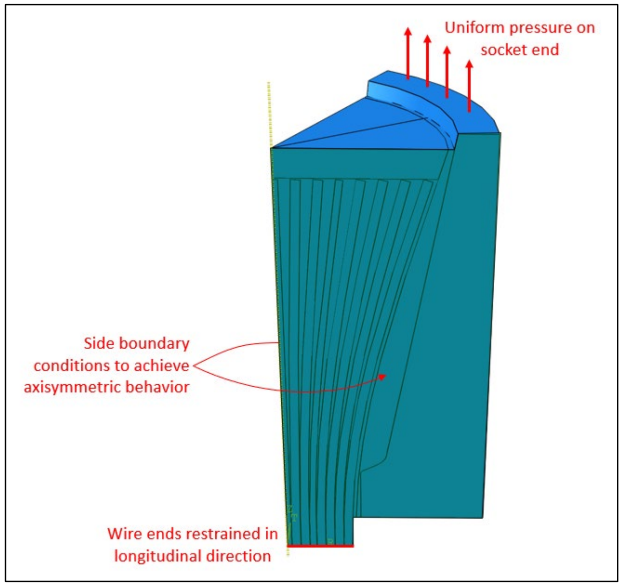

Figure 4: FE model loading and boundary conditions.

3.0 Analysis and Results

3.1 Baseline Model Behavior

The baseline model, whose wire broom is uniform, was first loaded to half of the cable's Minimum Breaking Strength and held at that load for 25 years to observe the evolution of the stress state in the socket casting over time.

The cone-shaped cavity of a spelter socket is expected to engage the zinc casting through wedging action, thus inducing a radial compressive stress in the casting. As shown in Figure 5, the radial stress in the zinc is close to zero when the socket is initially loaded. This is due to the casting bearing on the socket's shoulder instead of being wedged into the cone. However, a radial compressive stress gradually develops over time due to zinc flow. The zinc flow is driven by shear stress which, as shown in Figure 6, gradually decreases in the casting over time. When the shear stress is low, the casting is being held in the socket primarily through the intended wedging action. As shown in Figure 7, zinc flow also causes a stress redistribution among the cable's wires, with the outer wires picking up more tension than the inner wires. The cable slip after 25 years is negligible, at 0.07 inch. It is understood that even with a uniform wire broom, an actual zinc-filled spelter socket would likely exhibit a larger cable slip due to imperfections in the initial contact between the socket and zinc, which are not included in the FE model.

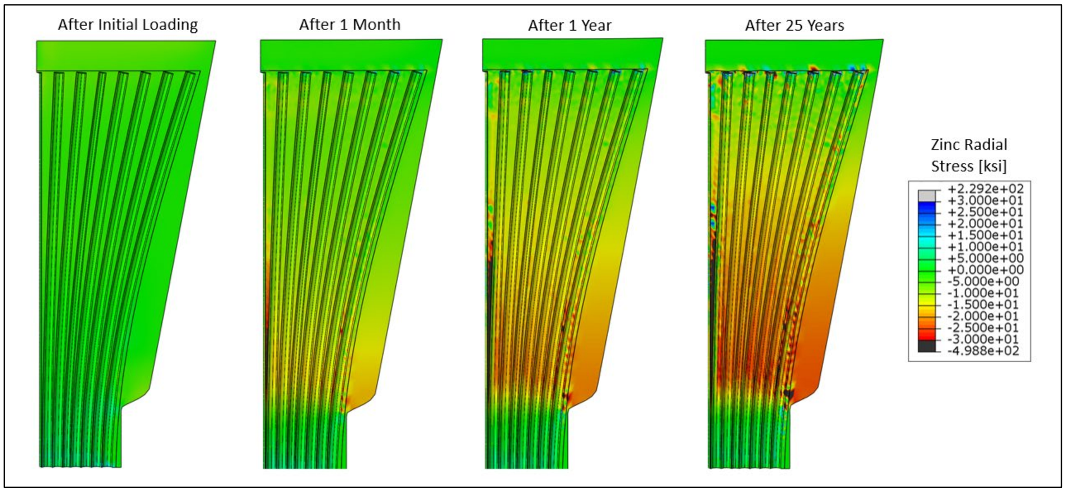

Figure 5: Radial stress evolution over time in zinc of baseline model.

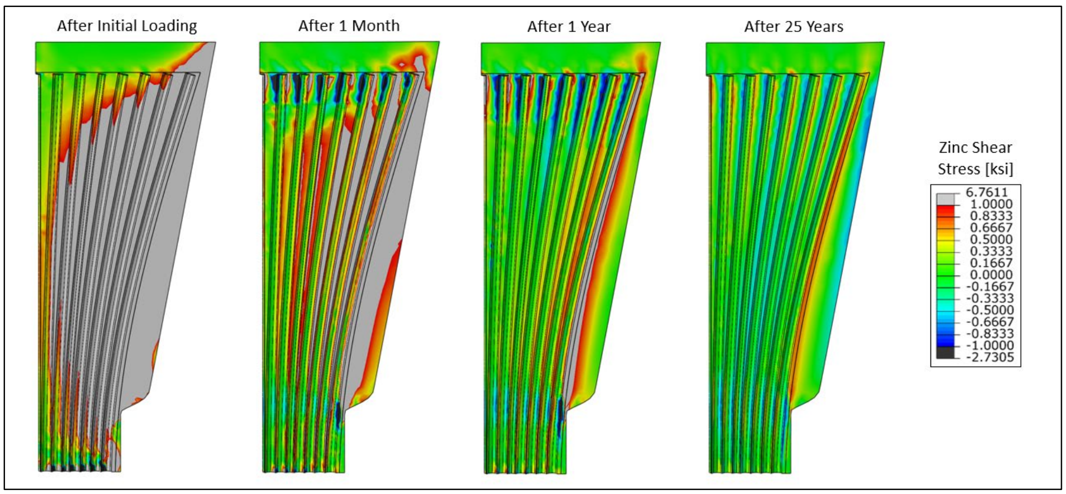

Figure 6: Shear stress evolution over time in zinc of baseline model.

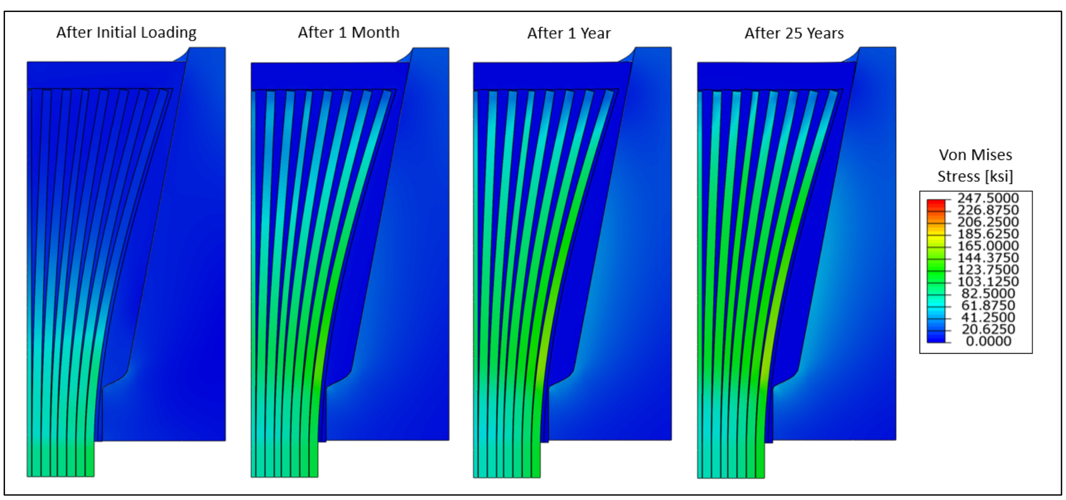

Figure 7: Von Mises stress evolution over time in wires of baseline model.

3.2 Effect of Socket Shoulder

The baseline model represents a socket design that includes a shoulder, which is a step in the profile of the socket's cavity near the front of the socket. Shoulders were present on most of the telescope's sockets, including the socket that experienced the largest cable slip (B12W_G) and where the first two cable failures occurred (M4N_T and M4-4_T). Zinc-filled spelter sockets, however, do not always have a shoulder. This was for example the case for the platform-end sockets of the telescope's auxiliary main cables. The presence of a shoulder can affect the load transfer mechanism between zinc casting and socket, as the casting can bear directly on the shoulder. Without a shoulder, the load transfer must occur through wedging action as the casting is squeezed in the cone-shaped cavity of the socket.

To study the effect of the shoulder on socket behavior, the FE model of a socket without a shoulder was derived from the baseline model by removing the shoulder (Figure 9, Figure 8). The front and back diameters of the socket's cavity is the same in both models, but the cavity's slope is steeper in the model with a shoulder. Both models were loaded to half of the cable's breaking strength, and the load was held for a year before comparing wire tension and zinc shear stress.

As shown in Figure 8, when the socket is initially loaded, the shoulder causes higher shear stress at the front of the zinc casting because the casting bears on the shoulder. However, after allowing the zinc to creep for a year, the shear stress distribution is similar in the models with and without a shoulder.

Similarly, the wire stress and cable slip are almost equivalent between the creep models with and without shoulders (Figure 9). The shoulder does not appear to have a significant impact on the long-term behavior of the socket.

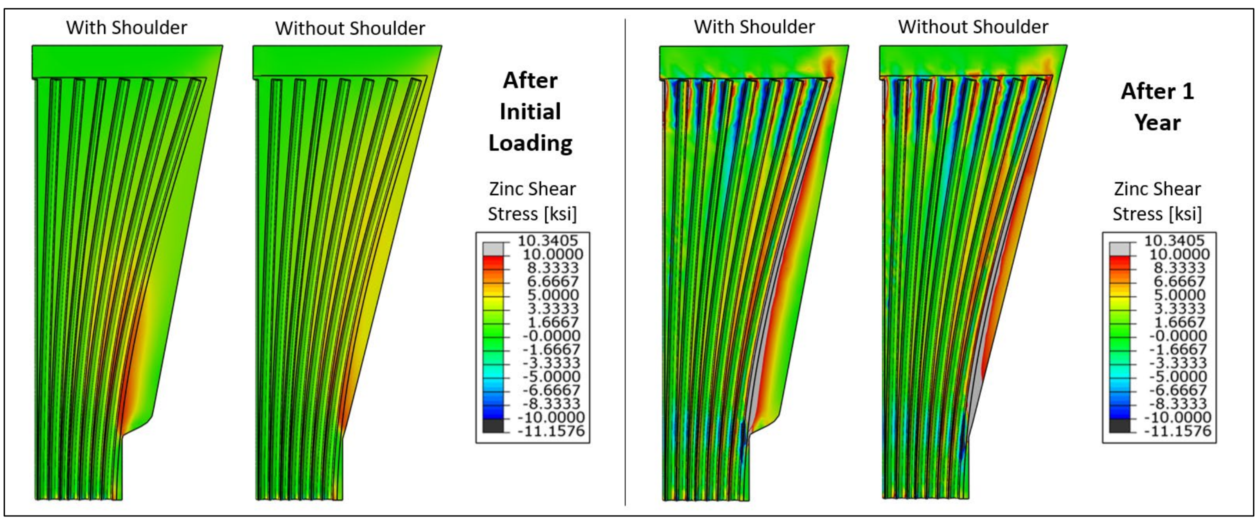

Figure 8: Zinc casting shear stress with and without shoulder.

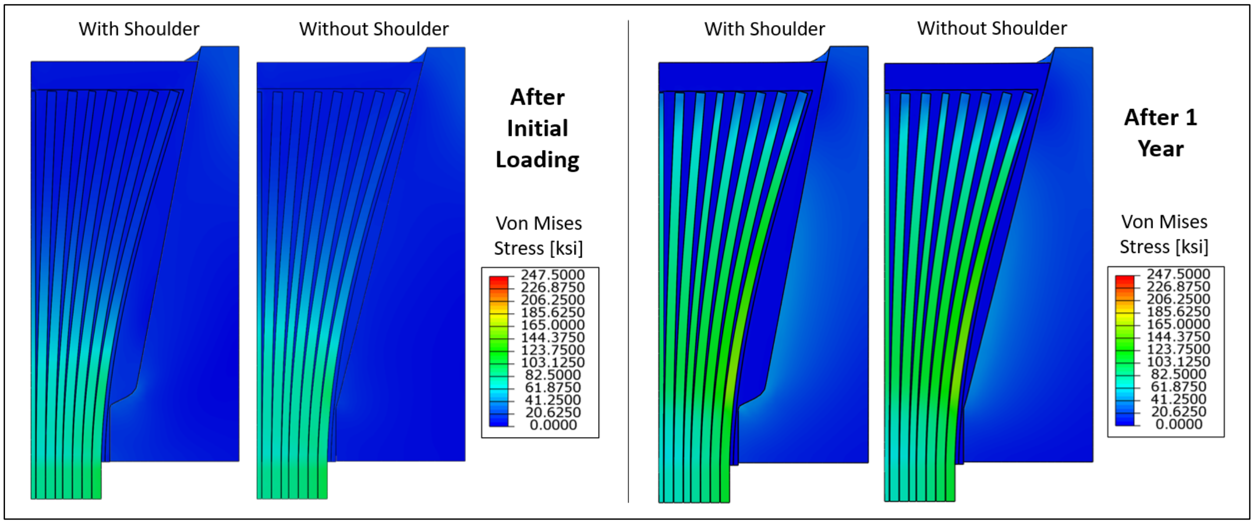

Figure 9: cable slip and wire stress with and without shoulder.

3.3 Effect of Wire Broom Extent

The cable wires are spread out inside the socket to fill the cone-shaped cavity and form a wire broom. The laboratory analysis of the socket samples indicates that the wire brooms vary between sockets, as they were shaped manually during the socket fabrication. Wire broom variations could cause or contribute to the different cable slips observed between sockets and the eventual failure of socket M4N_T. Therefore, we performed a series of FE analyses to investigate the effect of wire broom on socket behavior. As shown in Figure 10 to Figure 15, five wire brooms were considered.

The wire stresses and cable slips are equivalent in every broom when the socket is initially loaded (Figure 10), with the casting bearing on the shoulder. Zinc flow over time causes a stress redistribution resulting in the broomed-out wires carrying most of the cable's tension (Figure 11, Figure 12). In the socket with a single layer of wires broomed out, those wires rupture after just three years of zinc flow (Figure 11), causing failure of the socket. The sockets with at least two layers of wires broomed out do not fail after 25 years, and at that point the broomed-out layers carry most of the cable tension.

The shear stress in the zinc is also equivalent in every socket after the initial loading (Figure 13), and changes over time as zinc flows (Figure 14, Figure 15). The shear stress is higher within a zone located on the outer side of each wire layer, where the wires transfer their load to the zinc. For the socket with a single layer of wires broomed out, a broad zone of high shear stress is present around the central bundle of wires, which are not broomed out. This causes those un-broomed wires to gradually pull out of the socket, stretching and eventually rupturing the single layer of broomed-out wires.

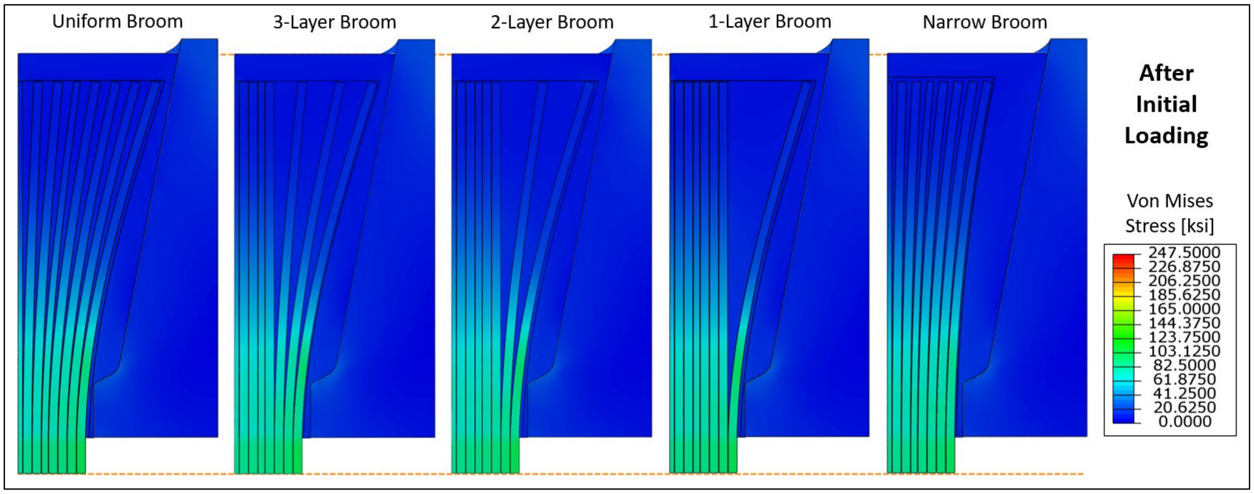

Figure 10: Wire tensile stress for different wire brooms after initial loading.

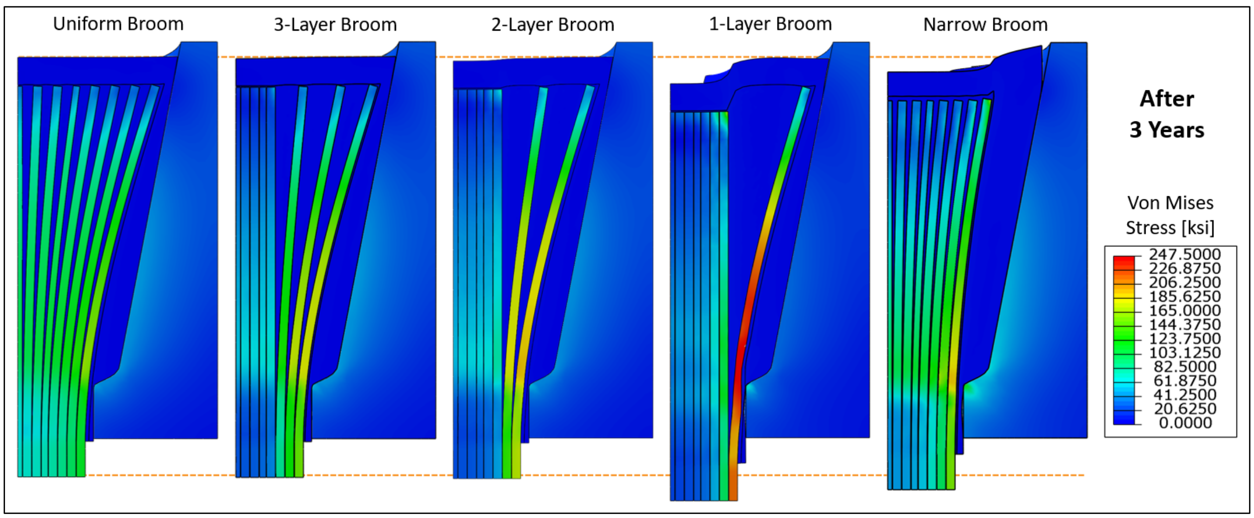

Figure 11: Wire tensile stress for different wire brooms after 3 years of zinc flow.

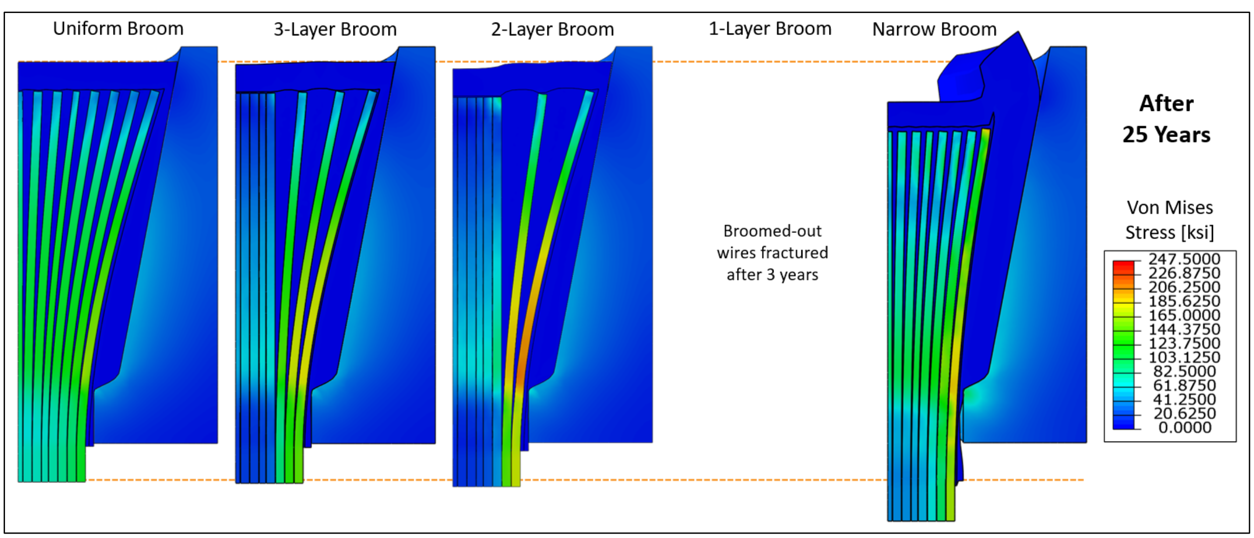

Figure 12: Wire tensile stress for different wire brooms after 25 years of zinc flow.

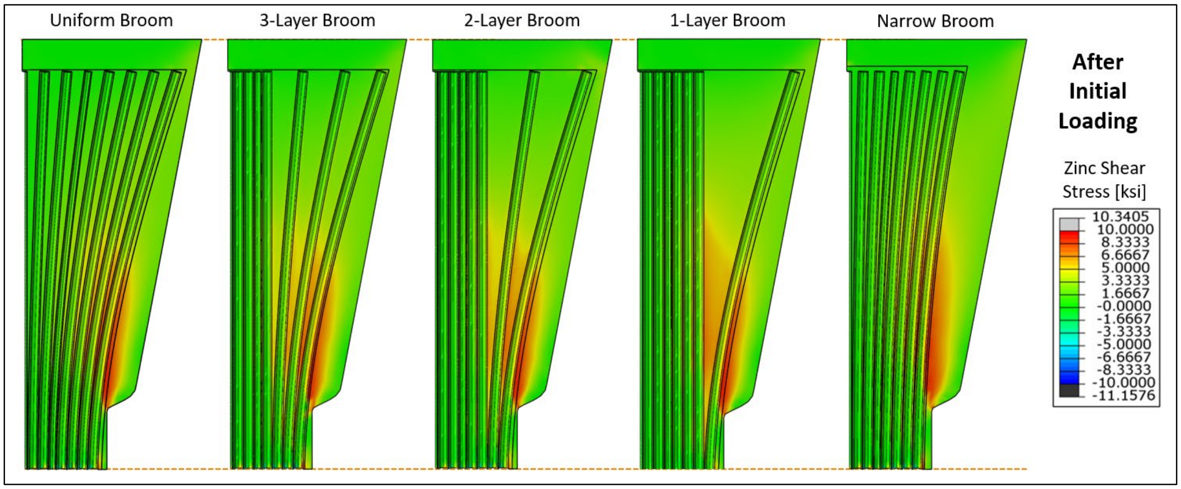

Figure 13: Zinc shear stress for different wire brooms after initial loading.

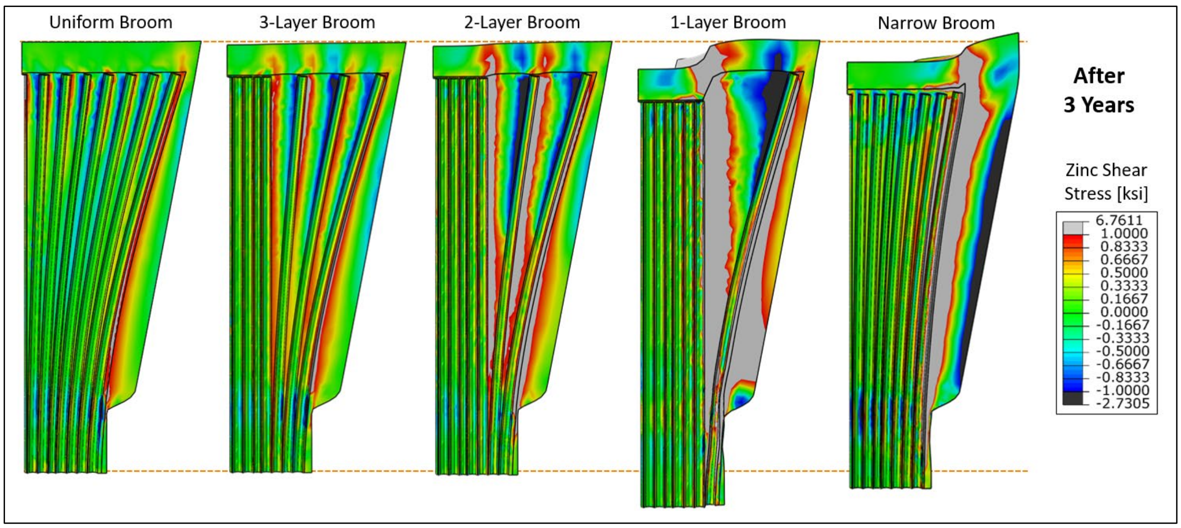

Figure 14: Zinc shear stress for different wire brooms after 3 years of zinc flow.

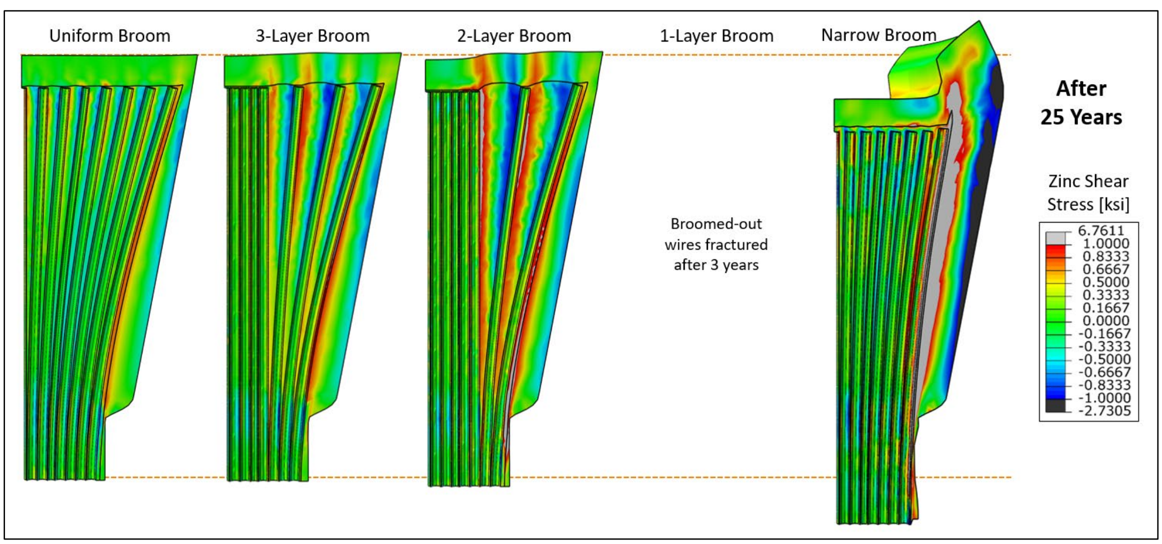

Figure 15: Zinc shear stress for different wire brooms after 25 years of zinc flow.