Appendix G
Cable Tensions during 2020 Events

1.0 Introduction

In the second half of 2020, the telescope experienced two cable failures approximately three months apart, and eventually collapsed three weeks after the second failure (Table 1). Between the two cable failures, the azimuth arm and Gregorian were moved to the stowed position, and the tiedowns were partially released to reduce the stress on the remaining cables, which were also surveyed to determine their current tensions. This appendix presents our analysis of the cable tensions in 2020, from the first cable failure to the structure's collapse.

Table 1: Timeline of 2020 cable failures and collapse.

Date	Event
8/10/2020	M4N fails during telescope operation. Telescope operation continues for 20 minutes before telescope is stopped.
8/11/2020	Azimuth arm and Gregorian are moved to stowed position. Tiedowns are released as much as possible.
8/28/2020 – 9/4/2020	Cable shapes are surveyed.
11/6/2020	M4-4 fails.
12/1/2020	M4-2 fails and triggers collapse of telescope.

2.0 August 2020 Sag Survey

After the first cable failure, the remaining cables were surveyed to determine the tension in each cable. The objectives at the time were to evaluate the stability of the structure and to assess potential repair options. We relied on the sag survey to calibrate our structural analysis models of the upgraded structure (Appendix F), since the survey provides a complete set of cable tensions (except for the failed cable M4N) at a specific time and for a known configuration of the telescope. Calibrating the models with measured tensions is important because the cable system is statically indeterminate.

2.1 Laser Scans

The sag survey consists of several laser scans performed by Arecibo Observatory (AO) between August 28 and September 4, 2020, after the telescope was placed in the stowed position and the tiedowns partially released following the first cable failure. The scans were performed on a few different days but within the same week and always at daytime. Based on our analysis of the effect of temperature on the cable tensions (Appendix I), the tension change that a cable may have experienced between scans is negligible compared to the actual cable tension. Collectively, the scans are therefore considered to be a single consistent survey of the cable system.

Each scan focuses on either the mains (Figure 1) or backstays (Figure 2) of a given tower. The scan data is a point cloud, with millions of points recreating the cable surfaces in three dimensions. Each scan was captured with the scanner set up at a fixed position. The scanner's accuracy decreases with distance, generating noisier data for the cable surfaces located further away from the scanner (Figure 3). When selecting points to measure the cable sag, we avoided the noisier surfaces when possible, and otherwise selected average points within the noise consistently along the cable.

Figure 1: Tower 8 mains in August 2020 laser scan.

Figure 2: Tower 12 backstays in August 2020 laser scan.

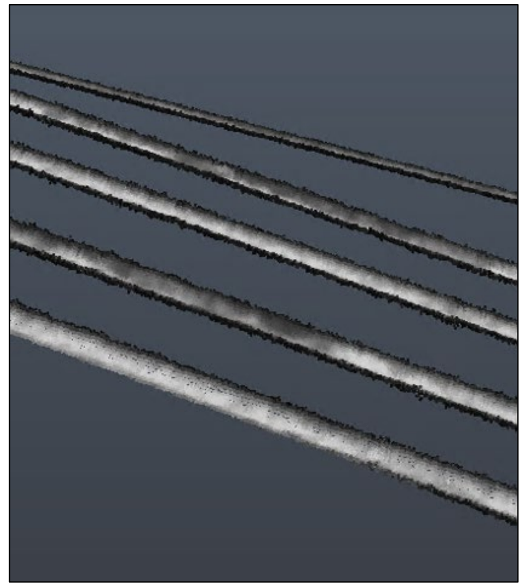

Figure 3: Tower 12 mains in August 2020 laser scan.

2.2 Cable Tensions

The only transverse load on any main or backstay is the cable's own weight, except for the Stockbridge vibration dampers. However, the dampers are relatively light and located near the ends of the cable, so their impact on the cable shape is negligible. Under its own weight, a cable assumes a catenary shape that is a function of the cable's weight and the horizontal component of the cable tension. Because this horizontal force is constant along the cable, by extracting the catenary shape from the sag survey and with the cable's weight known, we can calculate the horizontal force. The vertical force and the tension (axial force) can then be derived geometrically from the horizontal force and cable shape.

The procedure to determine a cable tension from the sag survey is illustrated in Figure 4. We first select approximately 10 points along the cable from the laser scan data, covering as much cable length as possible while staying away from the cable ends where data tends to be sparse or noisier. The missing or noisy data near the cable ends is due to the vibration dampers, obstruction from the terrain, and/or distance from the scanner. We then fit a catenary shape to the selected points, using the equation and parameters provided in Table 2. The only unknown parameter is the horizontal cable force, which is adjusted until the catenary best fits the selected points. The average distance between the selected points and the best-fit catenary is less than 0.25 inch for the mains and backstays, and less than 0.50 inch for the waveguide cables. Once the horizontal force in the cable is determined, the vertical reactions at both cable ends and the average tension (axial force) in the cable can be calculated, as shown in Table 2.

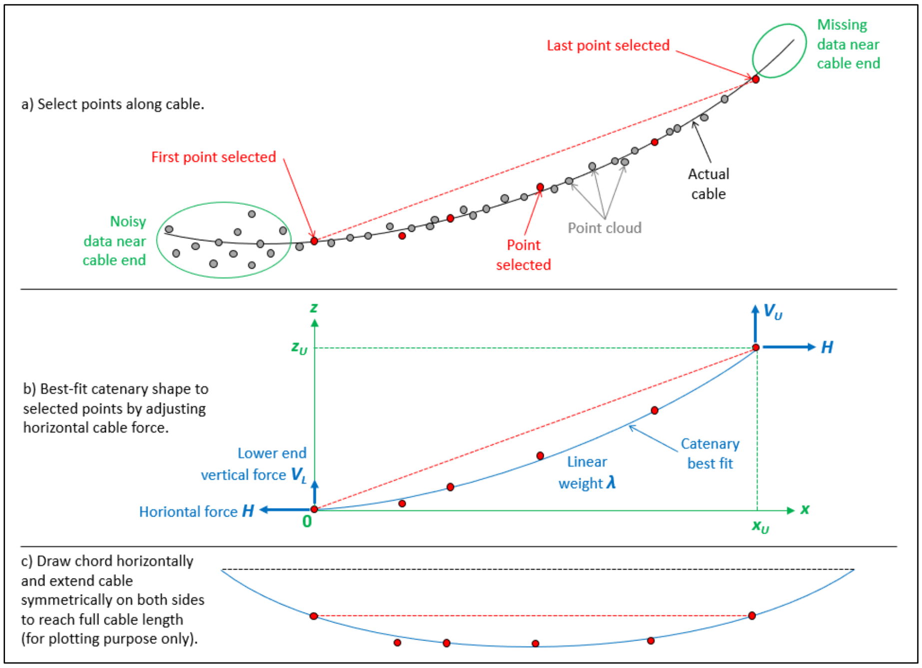

Figure 4: Sag measurement on laser scan data.

Table 2: Equations for catenary shape fitting and cable tension calculations.

Cable properties	H = horizontal force λ = linear weight (Table 3) x_u = horizontal length (Figure 4.b) z_u = vertical length (Figure 4.b)		
Catenary shape	$z = z_0 + a\cosh\left(\frac{x - x_0}{a}\right)$		
Catenary shape parameters	$a = \frac{H}{\lambda}$	$x_0 = \frac{1}{2}\left(x_u - 2a\, sinh\left(\frac{z_u}{2a\, sinh\left(\frac{x_u}{2a}\right)}\right)\right)$	$z_0 = -a\, cosh\left(\frac{x_0}{a}\right)$
Lower end vertical force	$V_L = sinh\left(-\frac{x_0}{a}\right)$		
Upper end vertical force	$V_U = sinh\left(\frac{x_u - x_0}{a}\right)$		
Average tension (axial force)	$F = \frac{1}{2}\left(\sqrt{H^2 + V_L{}^2} + \sqrt{H^2 + V_U{}^2}\right)$		

The selected points, fitted catenary, and average tension are shown in Figure 5 through Figure 9 for every cable analyzed. At each tower, to significant shape difference was observed among the four original mains and among the five original backstays. As a result, we analyzed a single cable from each set and considered the calculated cable tension applicable to every cable of the set. The assumptions and results of the cable tension calculations are provided in Table 3, Table 4 and Table 5.

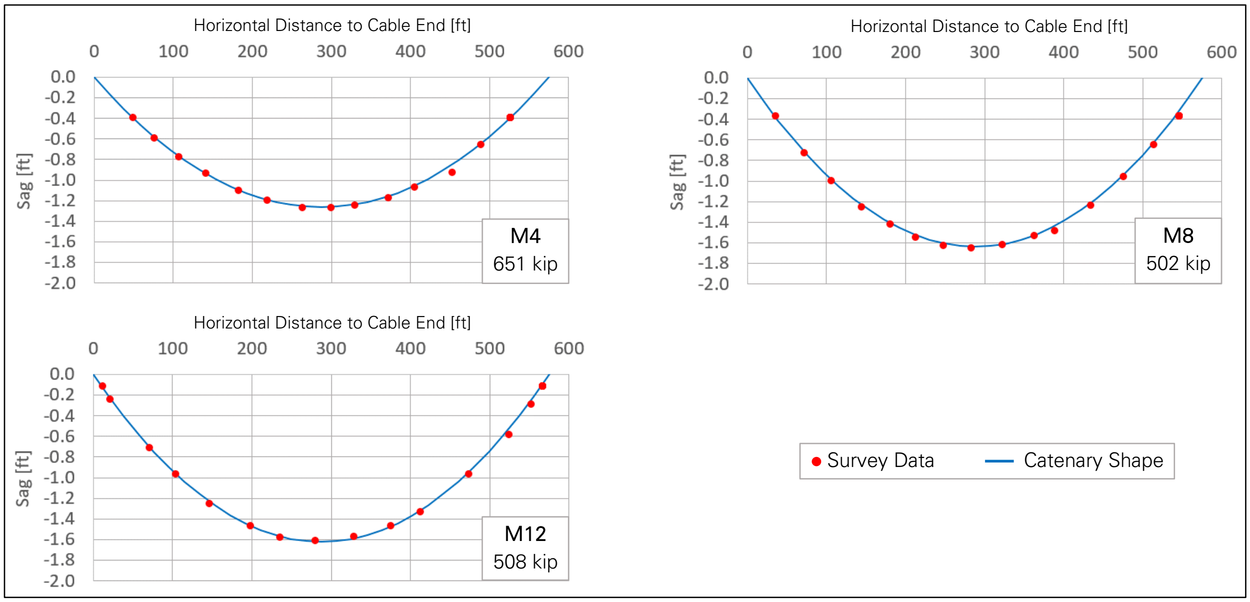

Figure 5: Catenary shape fitting on sag survey data for original main cables.

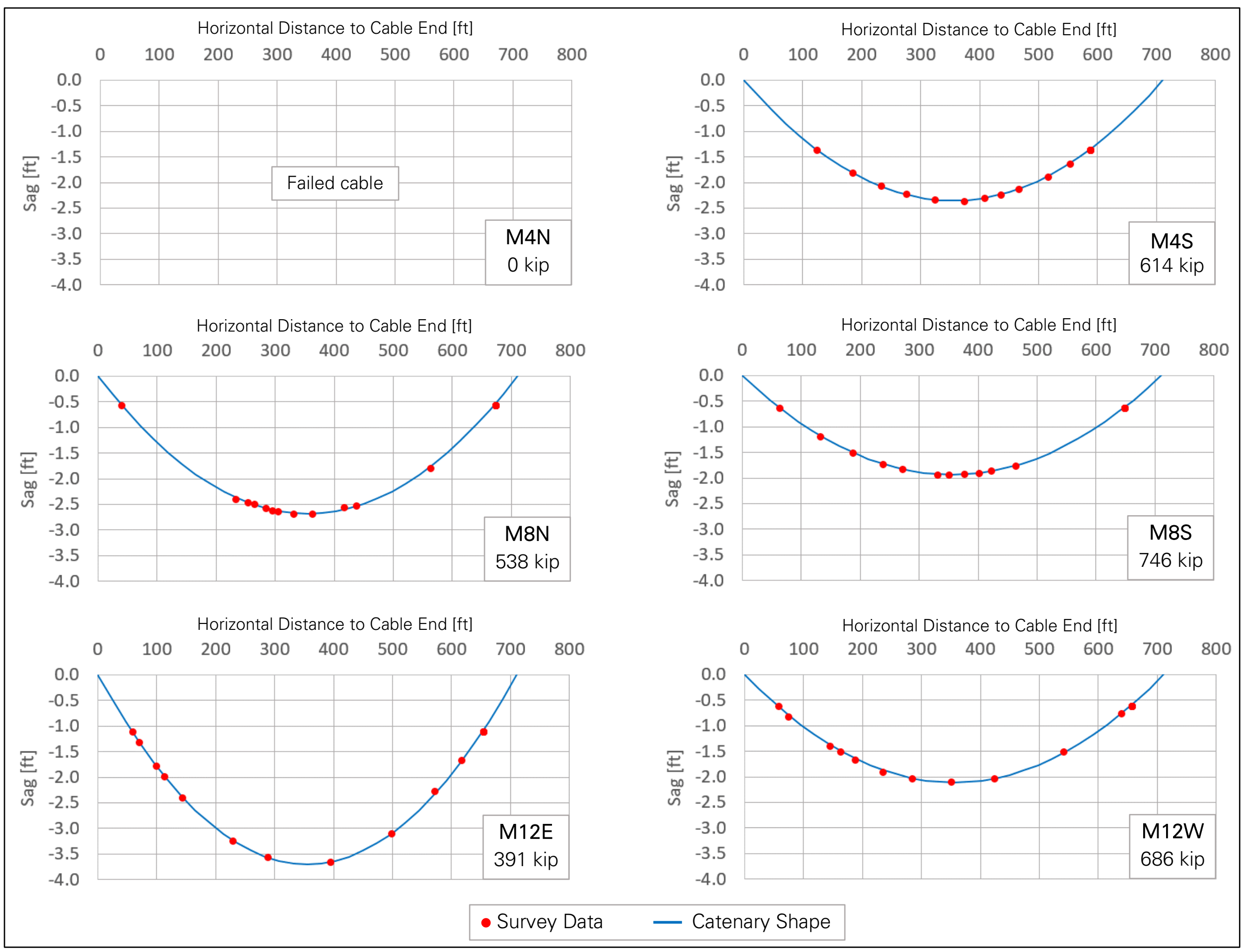

Figure 6: Catenary shape fitting on sag survey data for auxiliary main cables.

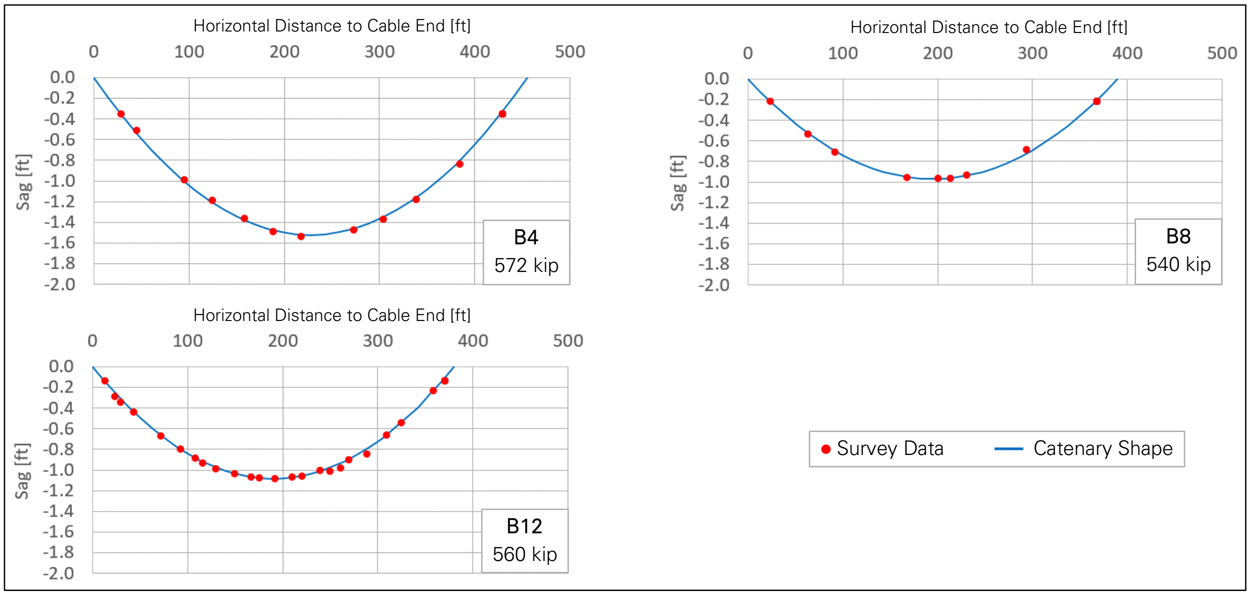

Figure 7: Catenary shape fitting on sag survey data for original backstay cables.

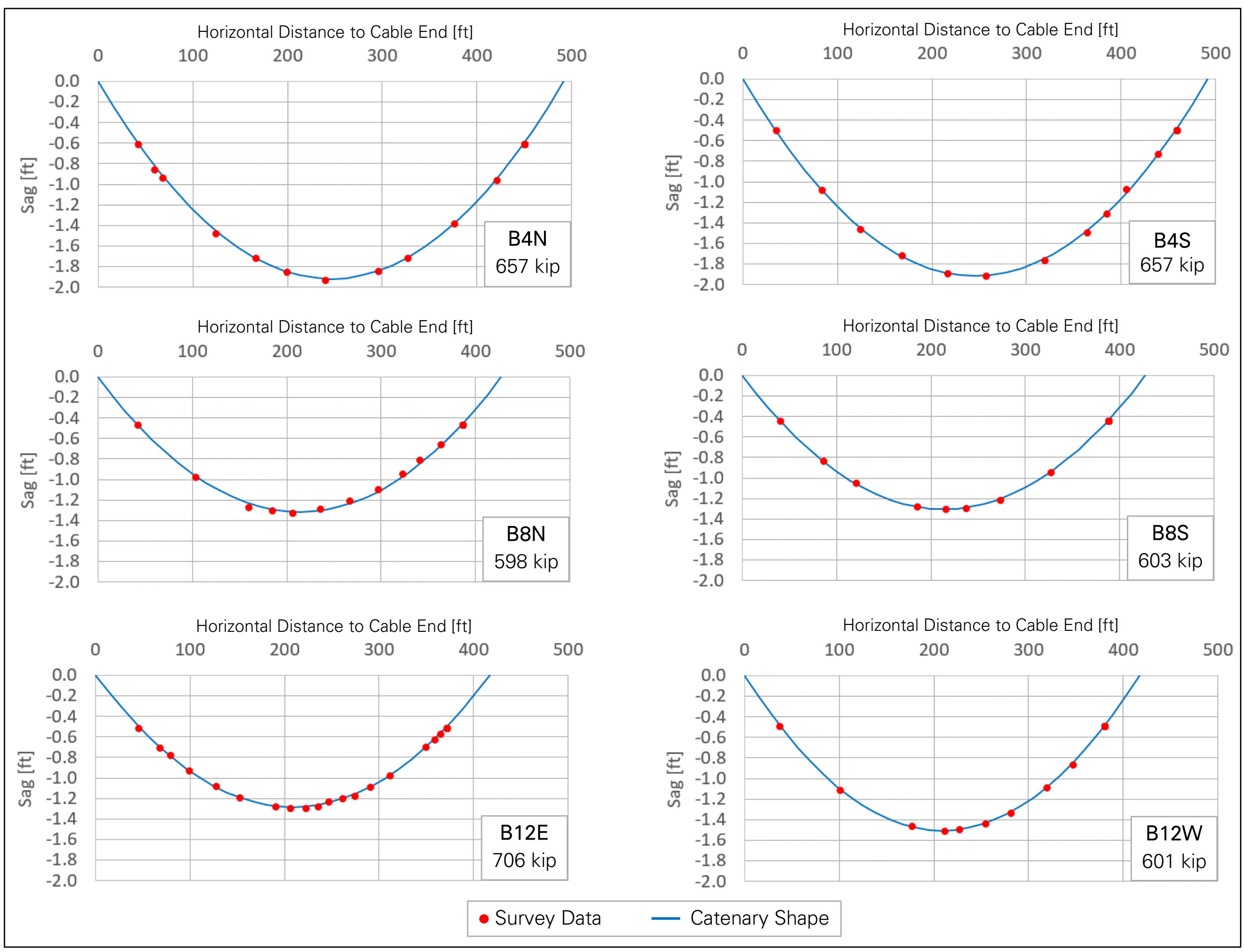

Figure 8: Catenary shape fitting on sag survey data for auxiliary backstay cables.

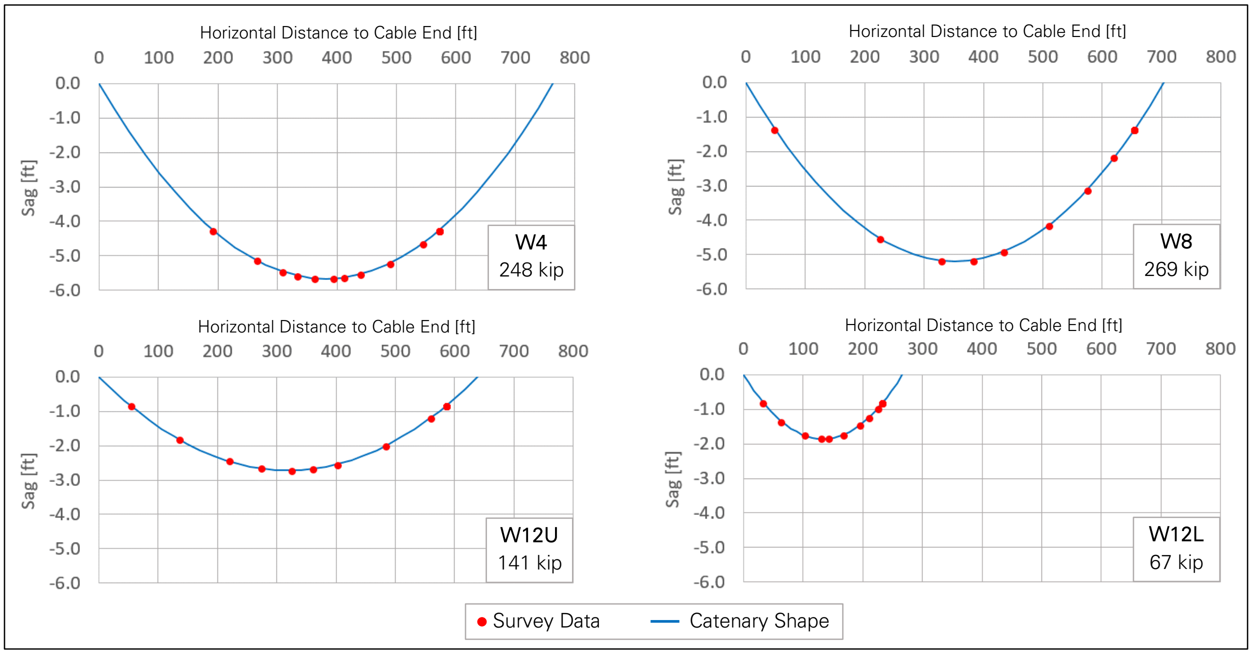

Figure 9: Catenary shape fitting on sag survey data for waveguide cables.

The suspended structure stabilized after the first cable failure, and therefore the resultant of the cable tensions acting on the platform must be zero in both horizontal directions. However, when summing up the horizontal forces derived from the sag survey, we obtain resultants of 72 kilopound (kip) and 15 kip in the south and west directions, respectively. This imbalance is small considering that each of the main cables carries several hundred kip, and may be due to temperature differences between the laser scans or noise in the laser scan data. Rather than introducing this imbalance in the structural analysis models, we applied small correction factors to the measured cable tensions until the horizontal resultant on the suspended structure was exactly zero. Correction factors were only applied to the main cables, and the same factor was applied to every main cable connected to a given tower. As shown in Table 3, the correction factors change the cable tensions by less than 1.5 percent (factors are between 0.985 and 1.015).

Table 3: Sag survey results and correction for main cables.

Cable	Qty.	Linear Weight [lbf/ft]	Distance Between Work Points			August 2020 Sag Survey		Force Correction Factor	Corrected Forces		
			North [ft]	West [ft]	Up [ft]	Sag [ft]	Avg. Tension [kip]		Avg. Tension [kip]	Horizontal [kip]	Vertical at Platform [kip]
M4	4	18.9	288	498	-130	1.26	651	0.991	645	629	137
M4N	1	22.2	418	575	-126	-	-	-	-	-	-
M4S	1	22.2	289	649	-126	2.35	614	0.991	609	599	98
M8	4	18.9	288	-498	-130	1.64	502	0.986	495	483	104
M8N	1	22.2	418	-575	-126	2.68	538	0.986	531	522	84
M8S	1	22.2	289	-649	-126	1.94	746	0.986	736	724	120
M12	4	18.9	-575	0	-130	1.62	508	1.012	514	501	108
M12E	1	22.2	-707	-74	-126	3.70	391	1.012	396	390	61
M12W	1	22.2	-707	74	-126	2.11	686	1.012	694	683	113

Table 4: Sag survey results for backstay cables.

Cable	Qty.	Linear Weight [lbf/ft]	Distance Between Work Points			August 2020 Sag Survey		
			North [ft]	West [ft]	Up [ft]	Sag [ft]	Avg. Tension [kip]	Horizontal Force [kip]
B4	5	22.2	-228	-394	-327	1.52	572	465
B4N	1	27.6	-232	-433	-351	1.92	657	535
B4S	1	27.6	-259	-418	-351	1.92	657	535
B8	5	22.2	-195	338	-192	0.97	540	485
B8N	1	27.6	-198	378	-213	1.32	598	535
B8S	1	27.6	-229	361	-213	1.30	603	540
B12	5	22.2	380	0	-273	1.08	560	455
B12E	1	27.6	417	-20	-297	1.28	706	575
B12W	1	27.6	416	20	-297	1.51	601	490

Table 5: Sag survey results for waveguide cables.

Cable	Qty.	Linear Weight [lbf/ft]	Distance Between Work Points			August 2020 Sag Survey		
			North [ft]	West [ft]	Up [ft]	Sag [ft]	Avg. Tension [kip]	Horizontal Force [kip]
W4	1	18.9	412	642	-91	5.66	248	246
W8	1	22.2	412	-570	-91	5.19	269	267
W12U	1	7.4	-638	36	-91	2.72	141	140
W12L	1	9.5	-263	42	-185	1.86	67	55

The resultant of the cable tensions on the suspended structure and three towers are shown in Table 6, before and after correction. At the top of the towers, the horizontal force is not zero, but less than 100 kip. This slight imbalance, compared to the magnitude of the cable tensions, can be explained by the M4N cable failure and the towers' ability to carry horizontal loads to the ground in bending. The final set of cable tensions derived from the sag survey is summarized in Figure 10.

Table 6: Resultant of cable tensions on suspended structure and towers.

Location	Direction	Resultant Force Before Correction of Main Tensions [kip]	Resultant Force After Correction of Main Tensions [kip]
Suspended structure	North	-72	0
	West	15	0
	Up	1,879	1,871
Top of Tower 4	Cable direction	-7	-36
	Perp. to cable direction	52	51
	Up	1,421	1,419
Top of Tower 8	Cable direction	-15	-61
	Perp. to cable direction	6	6
	Up	1,191	1,186
Top of Tower 12	Cable direction	-111	-74
	Perp. to cable direction	-43	-43
	Up	1,487	1,491

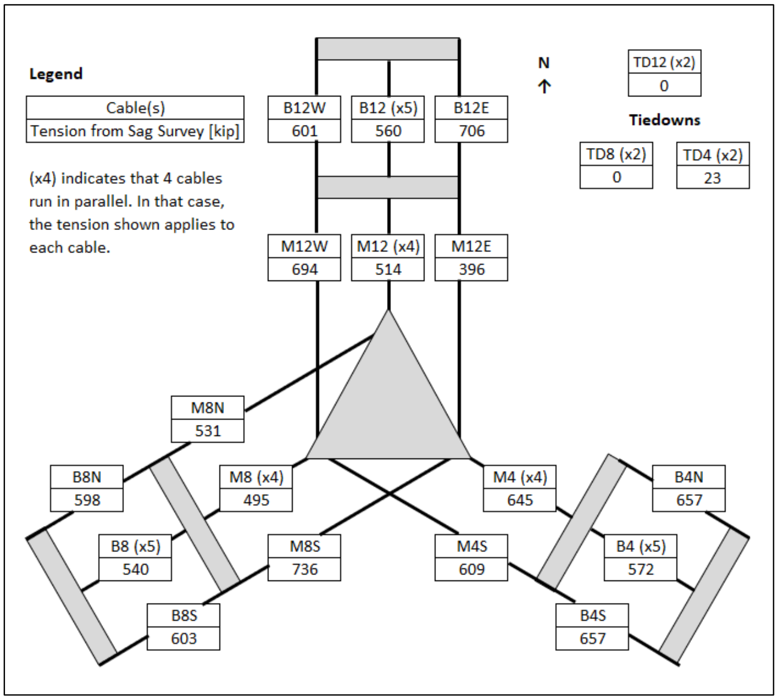

Figure 10: Cable tensions derived from August 2020 sag survey.

2.3 Suspended Structure Weight

The results of the sag survey indicate that the main cables carry a total vertical force of 1,871 kip (Table 6), which is balanced by the weight of the suspended structure and the tiedown tensions. The tiedown monitoring data provided by AO indicates that when the sag survey was performed, the average tension at daytime was 45 kip in tiedown 4, and zero in tiedowns 8 and 12. Tiedowns 8 and 12 were slack, as tiedown 12 became slack when the first cable failed, and tiedown 8 was released the next day to reduce the stress on the remaining cables. Tiedown 4 was also released as much as the jacking system would allow, but could not be fully slackened since the corresponding platform corner had shifted up after the cable failure. Considering the 45 kip tiedown tension, the weight of the suspended structure is 1,871 - 45 = 1,826 kip (Figure 11). The weight distribution between the steel trusses, the Gregorian and the line feed is detailed in Appendix F.

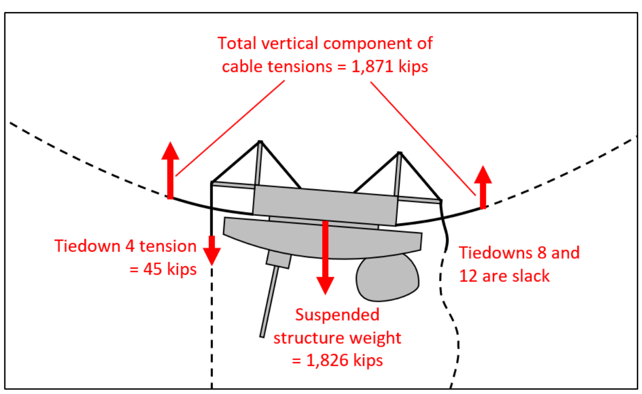

Figure 11: Suspended structure equilibrium during sag survey.

3.0 Cable Tensions before First Cable Failure

The August 2020 sag survey (section 2.0 above) provides a reliable measurement of the cable tensions after the failure of cable M4N (first cable failure). Starting from these known tensions, we performed a series of analyses to determine the cable tensions before failure, and in particular the tension in M4N when it failed.

Since cable M4N is connected to the east side of the platform near corner 12, its failure caused corner 12 to drop down and move west. During telescope operation, the platform corner positions were continuously monitored with laser rangers to adjust the tiedown tensions in order to keep the platform at a constant elevation through the day-night temperature cycles. Plots of the laser ranger data before and after the M4N failure were provided by AO and are shown in Figure 12. The plots show the elevation of the three platform corners and the rotation of the platform about three orthogonal axes. The fluctuations before failure are due to the telescope's operation since the tiedowns compensated for temperature effects. After failure, the fluctuations are due to temperature since the telescope and tiedown jacks were no longer operating. Table 7 summarizes the average elevations and rotations before failure, and the nighttime elevations and rotations after failure. We considered the nighttime values after failure because they are more consistent between days than the daytime values.

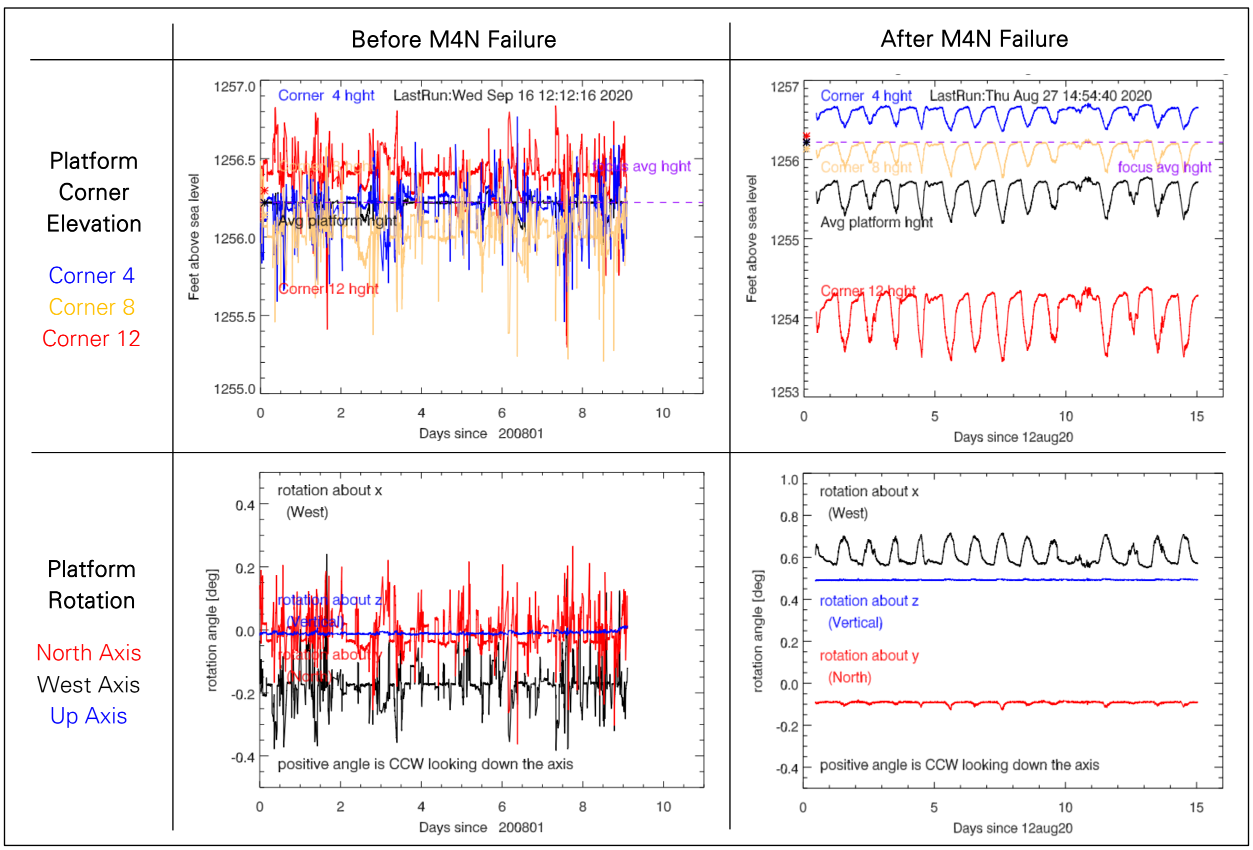

Figure 12: Platform corner elevation and rotation before and after M4N failure
(images: NAIC Arecibo Observatory, a facility of the NSF).

Table 7: Effect of M4N failure on platform position based on monitoring data.

	Average Before M4N Failure	Nighttime Average in Stowed Position After M4N Failure	Difference
Corner 4 Elevation [ft]	1256.1	1256.7	+0.6
Corner 8 Elevation [ft]	1256.1	1256.2	+0.1
Corner 12 Elevation [ft]	1256.4	1254.3	-2.1
Rotation about North Axis [deg]	-0.02	-0.09	-0.07
Rotation about West Axis [deg]	-0.17	0.56	+0.73
Rotation about Up Axis [deg]	-0.01	0.49	+0.50

To determine the cable tensions before the M4N failure, we performed a series of analyses on the telescope model. Each analysis involves four steps representing different states of the structure in reverse chronological order. The parameters of each step are detailed in Table 8 and rely on the telescope monitoring data provided by AO and shown in Figure 17 (section 4.0 below).

In Step 1, the model is set up to match the state of the structure during the August 2020 sag survey, with M4N removed from the model and the other cable tensions as determined from the survey (section 2.0 above). Step 2 increases the tiedown tensions to represent the nighttime condition, as the sag surveys were performed at daytime. In Step 3, the azimuth arm and Gregorian are moved to the position they were in when M4N failed, and the M4N cable is added back to the model and tensioned to an assumed value, which represents the state of the structure just before the M4N failure. Finally, in Step 4, the azimuth arm and Gregorian are moved back to the stowed position, and the tiedowns are fully released. This represents a baseline state of the structure before the M4N failure, which we will use as a starting point for the analysis of the upgraded structure under environmental and operation loads (Appendix H, I, J and K).

Steps 3 and 4 were repeated with increasing values of the M4N tension assumed at the time of failure, from 400 kip to 800 kip in 25 kip increments. After each analysis with a different M4N tension, we output the change in corner elevation and platform rotation from Step 3 to Step 2, as illustrated in Figure 13. For these analyses, we modeled the azimuth arm as loads on the platform's ring girder and varied these loads to represent different positions of the azimuth arm and Gregorian.

Table 8: Analysis steps to determine cable tensions before failure M4N failure.

Step	1	2	3	4
State	Telescope stowed after M4N failure, daytime	Telescope stowed after M4N failure, nighttime	Just before M4N failure	Baseline = telescope stowed before M4N failure, tiedowns slack
Azimuth Arm Angle [deg]	258	258	30.0	258
Gregorian Angle [deg]	8.5	8.5	10.7	8.5
Line Feed Angle [deg]	8.8	8.8	8.8	8.8
Tiedown 4 Tension [kip]	45	50	37	0
Tiedown 8 Tension [kip]	0 (slack)	10	35	0
Tiedown 12 Tension [kip]	0 (slack)	0 (slack)	20	0
M4N Tension	0 (failed)	0 (failed)	Assumed	Determined through analysis continuing from Step 3
Other Cable Tensions	Derived from August 2020 sag survey	Determined through analysis continuing from Step 1	Determined through analysis continuing from Step 2	Determined through analysis continuing from Step 3

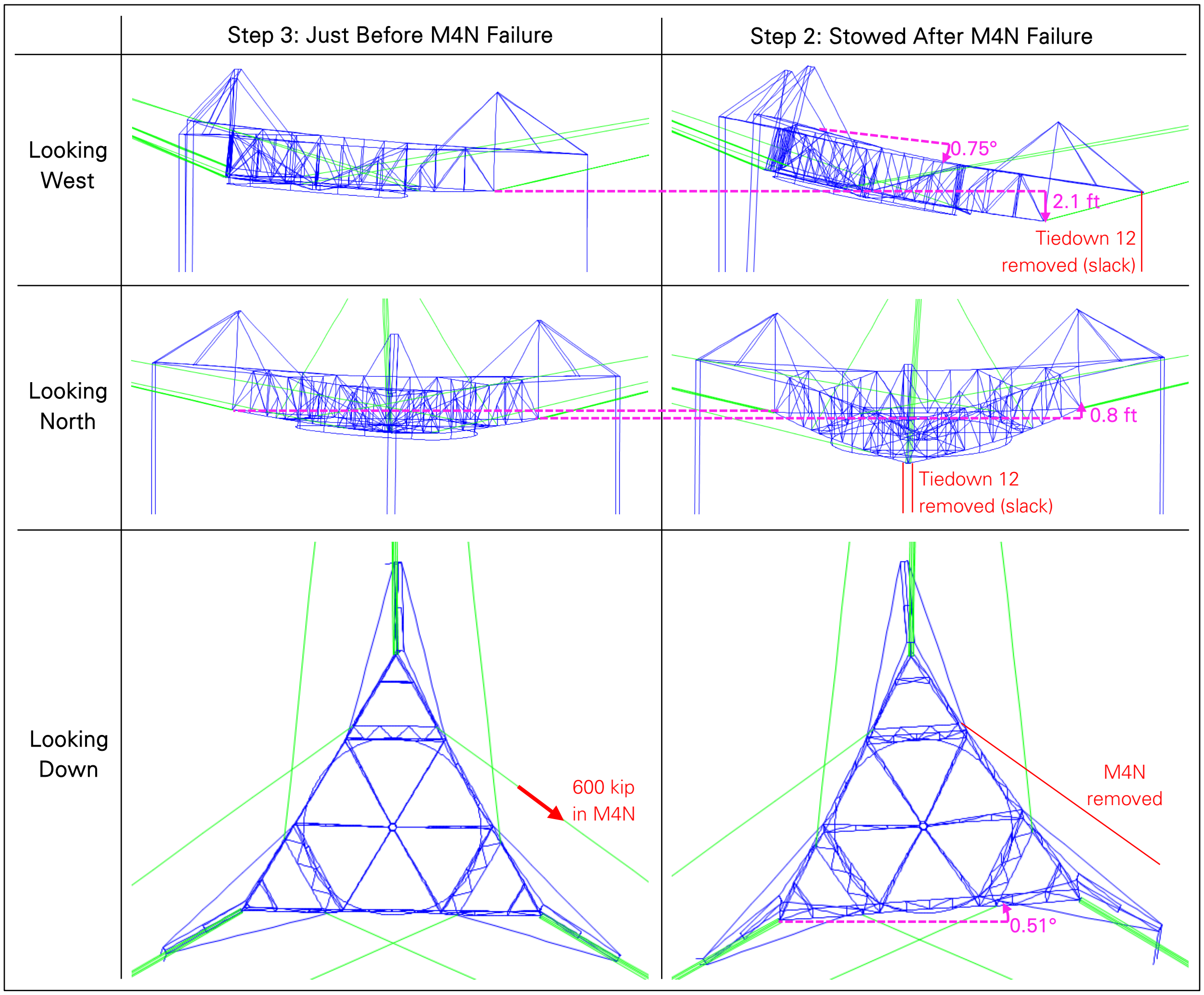

Figure 13: Analysis of platform response to M4N failure assuming M4N carries 600 kip at failure time. (Displacements magnified x10. Azimuth arm, Gregorian and line feed modeled as loads on ring girder.)

The results of the series of analyses are plotted in Figure 14 and compared to the monitoring data. The more significant changes observed in the monitoring data after the M4N failure are the rotation about the vertical axis (0.5 degree) and the drop of Corner 12 (2.1 feet), which results in a rotation about the west axis (0.73 degree). On the other hand, the rotation about the north axis (0.07 degree) and the vertical movement of Corner 4 (0.6 foot) and Corner 8 (0.1 foot) are less significant. For the three more significant parameters, the model matches the monitoring data when the M4N tension is between 580 kip and 600 kip at the time of failure.

Based on these results, we assume that M4N carried a tension of 600 kip when it failed. The complete set of cable tensions at the time of the M4N failure is provided in Figure 15 and compared to each cable's minimum breaking strength. The 600 kip in M4N are 46 percent of the cable's minimum breaking strength.

The cable tensions when the telescope is stowed with tiedowns released before the M4N failure are provided in Figure 16. These are the resultant tensions obtained in Step 4 of the analysis when the M4N tensions in Step 3 is 600 kip. With M4N still connected and the tiedowns released, these tensions correspond to a minimum stress state in the overall cable system.

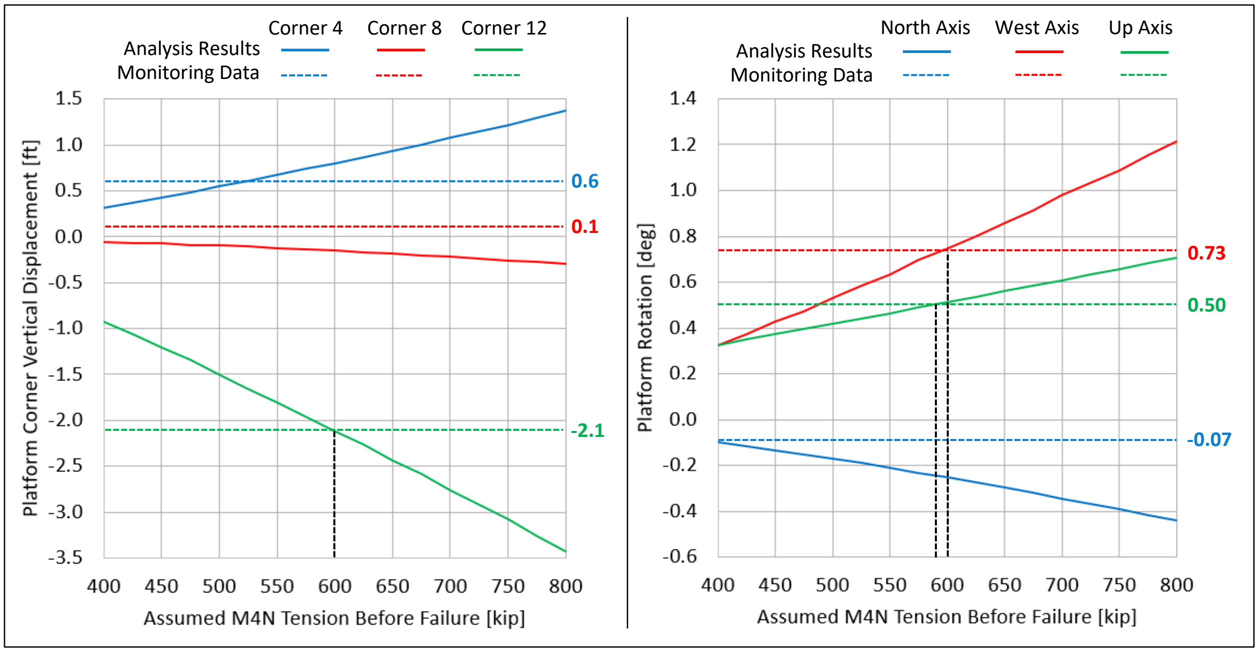

Figure 14: Determination of M4N tension just before failure from analysis results and monitoring data.

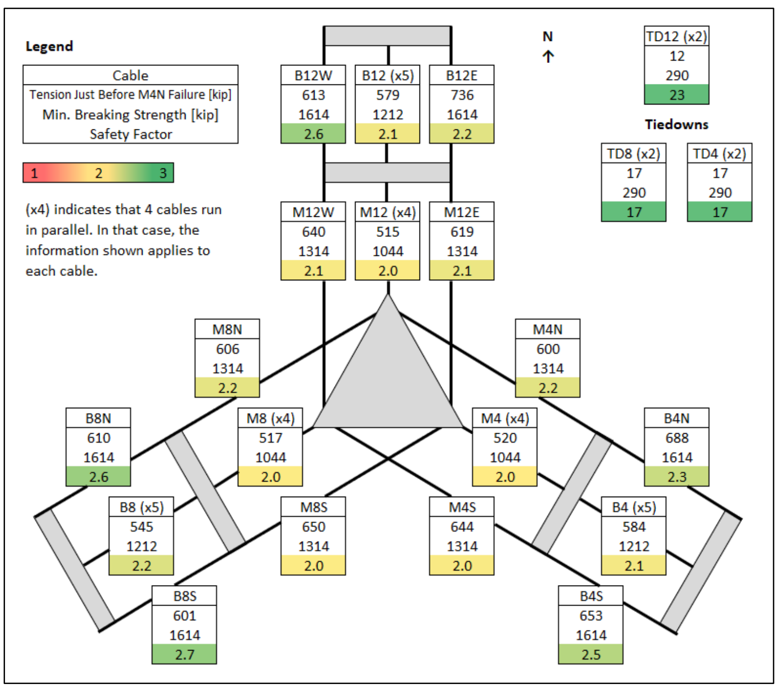

Figure 15: Cable tensions and safety factors just before M4N failure.

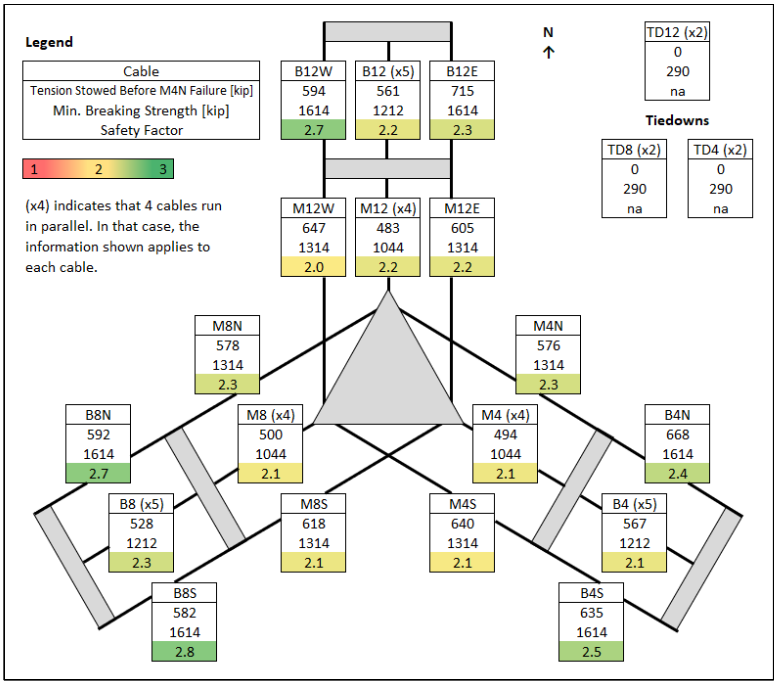

Figure 16: Cable tensions and safety factors with telescope stowed and tiedowns slack before M4N failure.

4.0 Cable Tension History after First Cable Failure

The telescope stopped operating shortly after the M4N failure (first cable failure). Still, the tensions in the cable system continued to change as the telescope was moved to the stowed position, and again when M4-4 failed (second cable failure). We performed an analysis of these events to determine the tensions experienced by each cable between the first cable failure and the collapse of the structure.

4.1 Timeline

The telescope monitoring data provided by AO documents what the structure underwent in the few days following the failure of M4N (Figure 17). Cable M4N failed at 2:35 AM on August 10, 2020, causing Tiedown 12 to go slack while increasing the tension in Tiedown 4. The telescope continued to operate for 20 minutes after the failure, with the azimuth arm rotating by 17 degrees and the Gregorian moving outwards by 2.5 degrees. When telescope operation was stopped after 20 minutes, the Gregorian was still located in the northeast quadrant that cable M4N had contributed support. On August 11, 2020, AO fully extended the jacks at the base of Tiedown 4 and Tiedown 8 to reduce the stress on the remaining cables. Later that day, the azimuth arm and Gregorian were moved to the stowed position. After this move, no adjustment was made to the telescope position until the collapse on December 1, 2020.

As the damaged structure experienced day-night temperature cycles, the tension varied from 40 to 50 kip in Tiedown 4 and from 0 to 10 kip in Tiedown 8, while Tiedown 12 remained slack.

On November 6, 2020, cable M4-4 failed near its tower-end socket. On December 1, 2020, cable M4-2 also failed near its tower-end socket, immediately followed by the remaining M4 cables and triggering the collapse of the telescope.

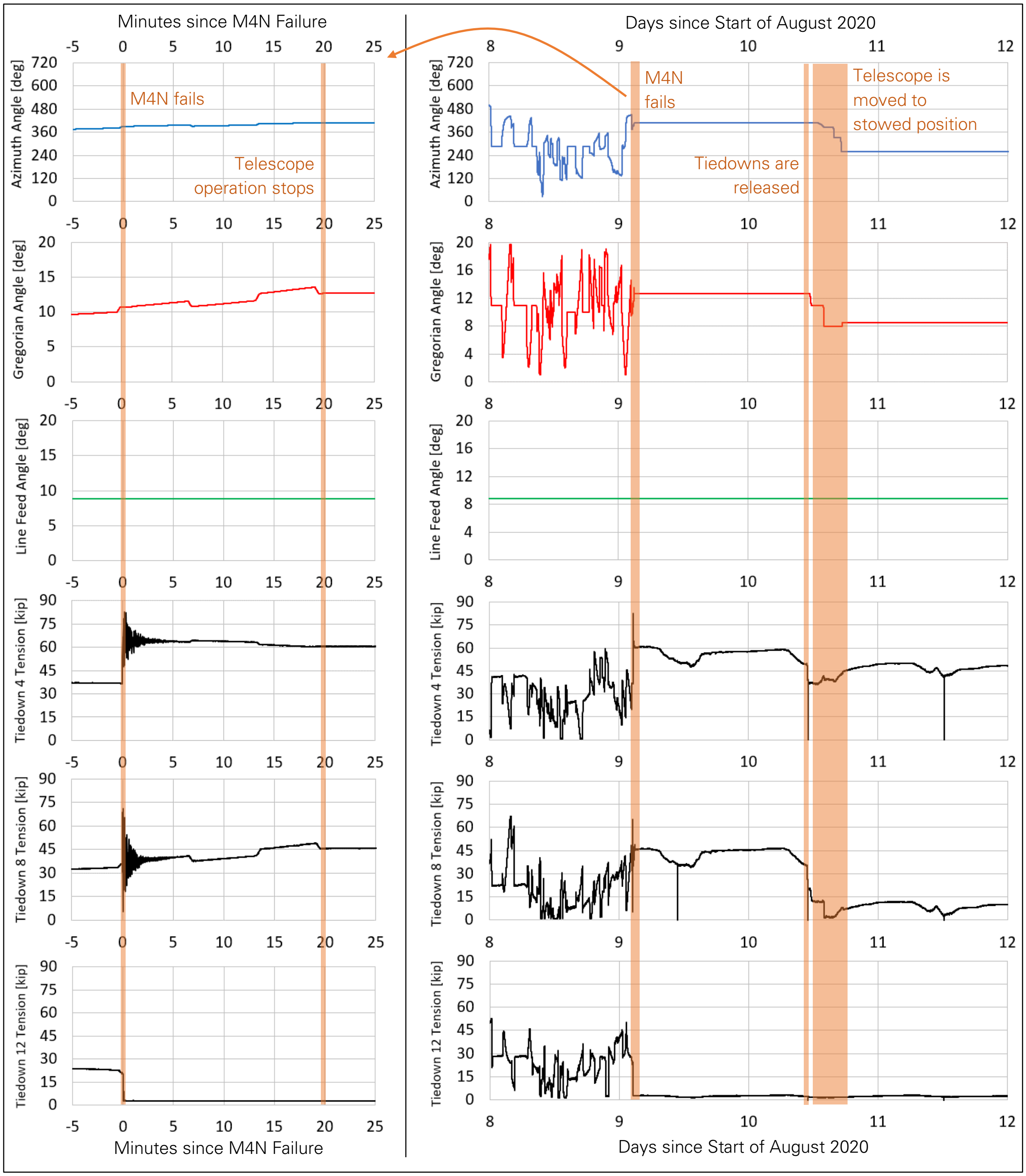

Figure 17: Telescope position and tiedown tensions before and after first cable failure (M4N).

4.2 Analysis

To determine the tensions in the cable system after the first cable failure, we performed a multi-step analysis that simulates the 2020 sequence of events in chronological order. At the start of the analysis, the model is set up to match the state of the structure just before the M4N failure (section 3.0 above). The subsequent steps simulate the M4N failure, the tiedown releases, the azimuth arm and Gregorian movement, and the M4-4 and M4-2 failures. The cable tension results are shown in Figure 18 for the mains and in Figure 19 for the backstays. The last step indicates what the cable tensions would have been if the structure had stabilized after the M4-2 failure on December 1, 2020. In reality the structure was never in that state, as the telescope collapsed immediately after the M4-2 failure.

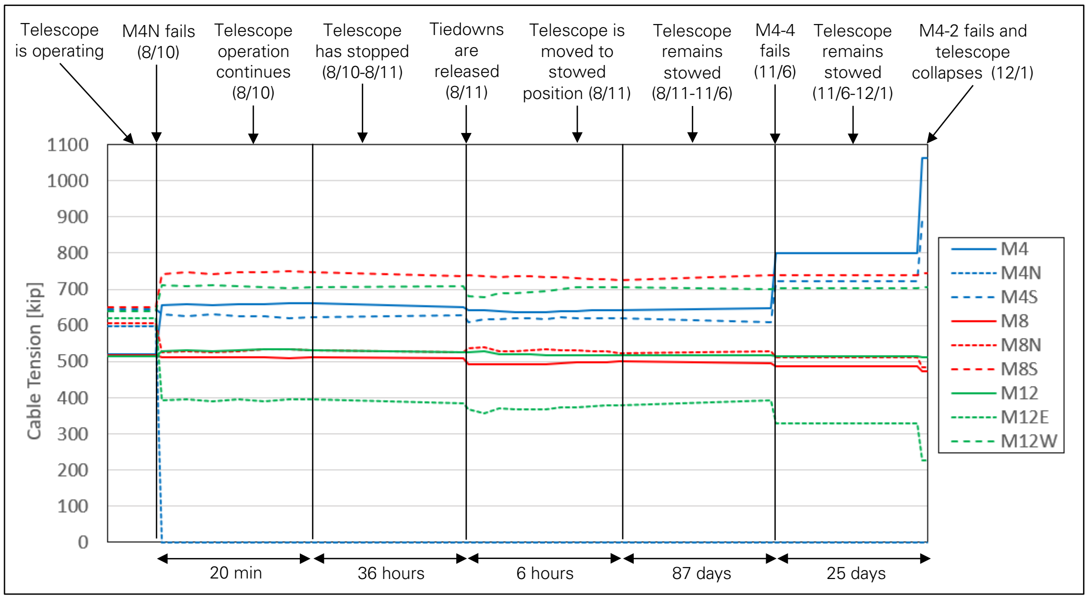

Figure 18: Main cable tensions from M4N failure to collapse.

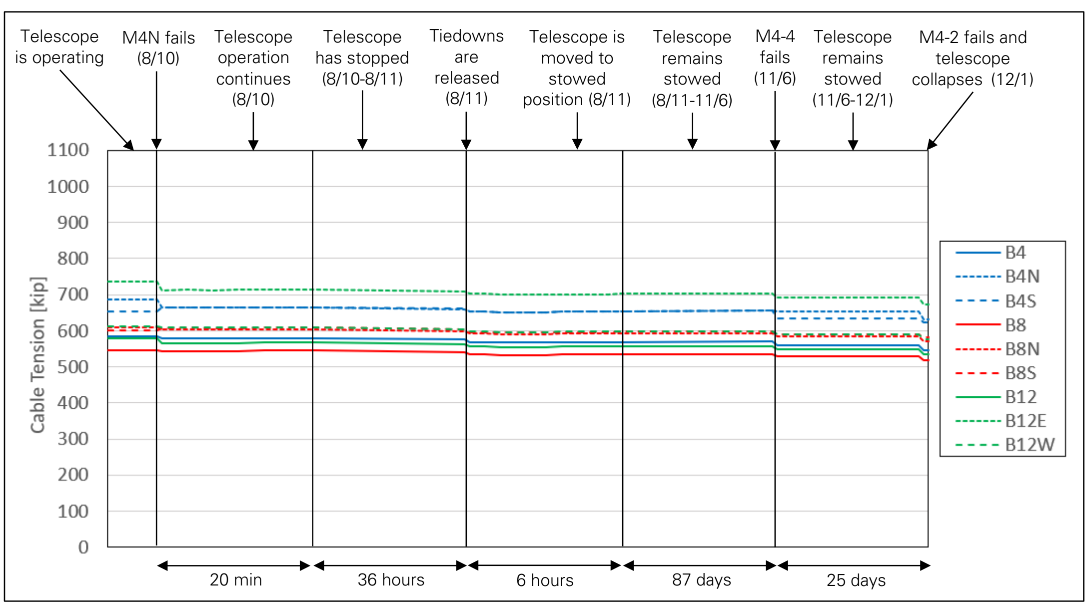

Figure 19: Backstay cable tensions from M4N failure to collapse.

Figure 20 through Figure 25 indicate the cable safety factor during and after each cable failure. The safety factor is the ratio of the cable's minimum breaking strength to the actual cable tension. The tensions during each cable failure are estimated, assuming a dynamic amplification factor of 2.0 due to the sudden cable tension release. The safety factor in the M4 cables was 1.6 and 1.3 after the M4N and M4-4 failures, respectively, and would have been 0.98 if the structure had withstood the M4-2 failure.

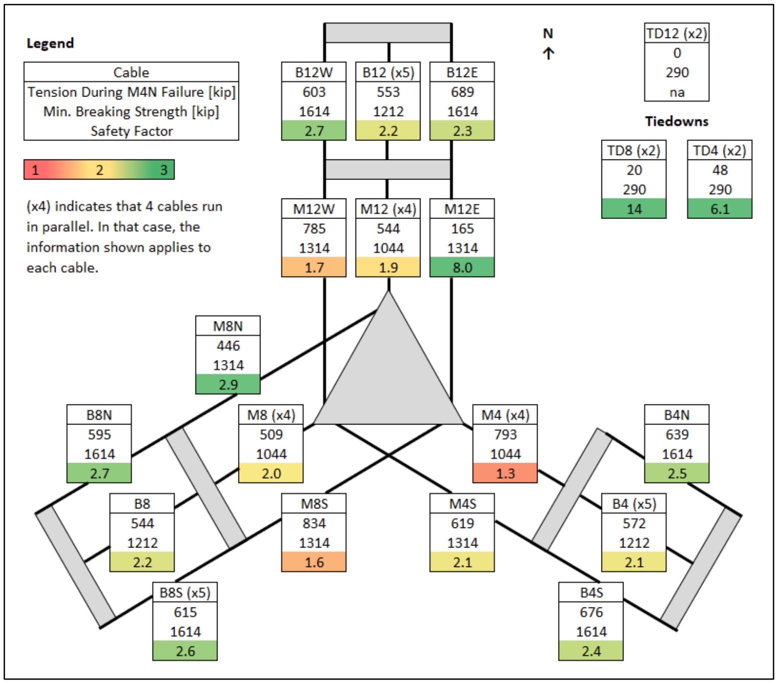

Figure 20: Cable tensions and safety factors during M4N failure, with 2.0 dynamic amplification.

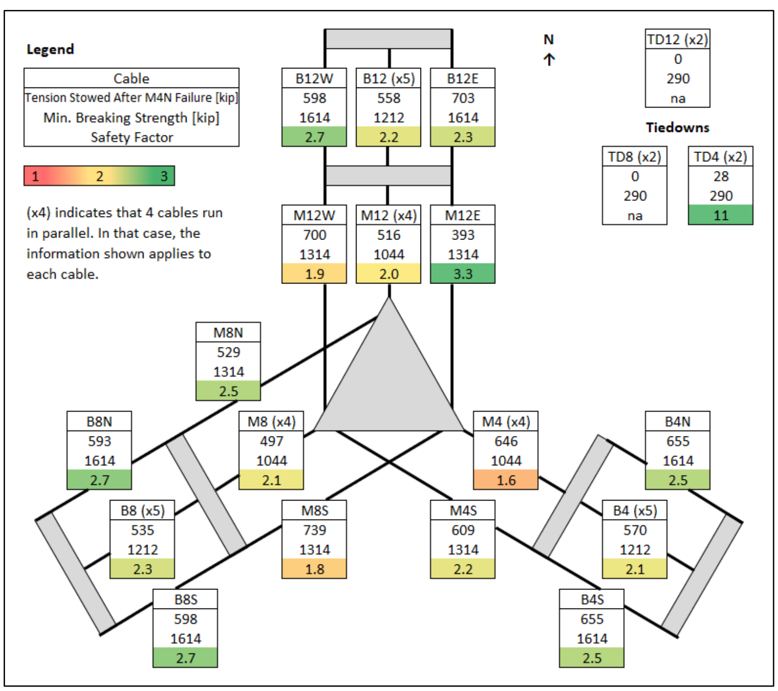

Figure 21: Cable tensions and safety factors when telescope is stowed after M4N failure.

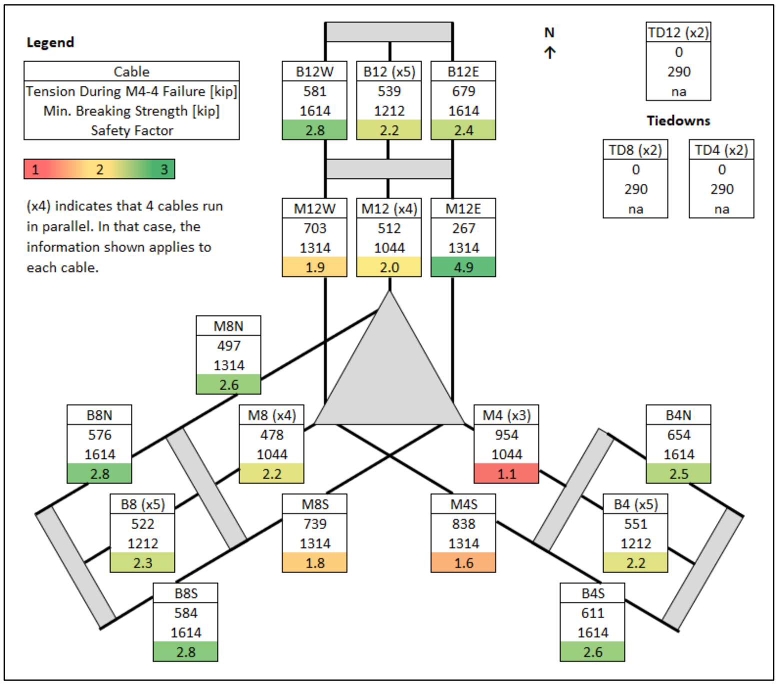

Figure 22: Cable tensions and safety factors during M4-4 failure, with 2.0 dynamic amplification.

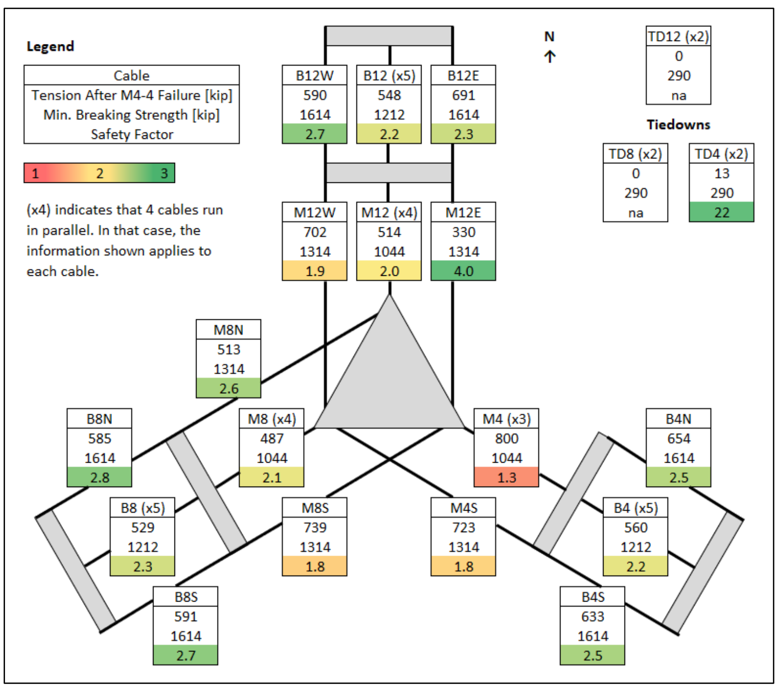

Figure 23: Cable tensions and safety factors after M4-4 failure.

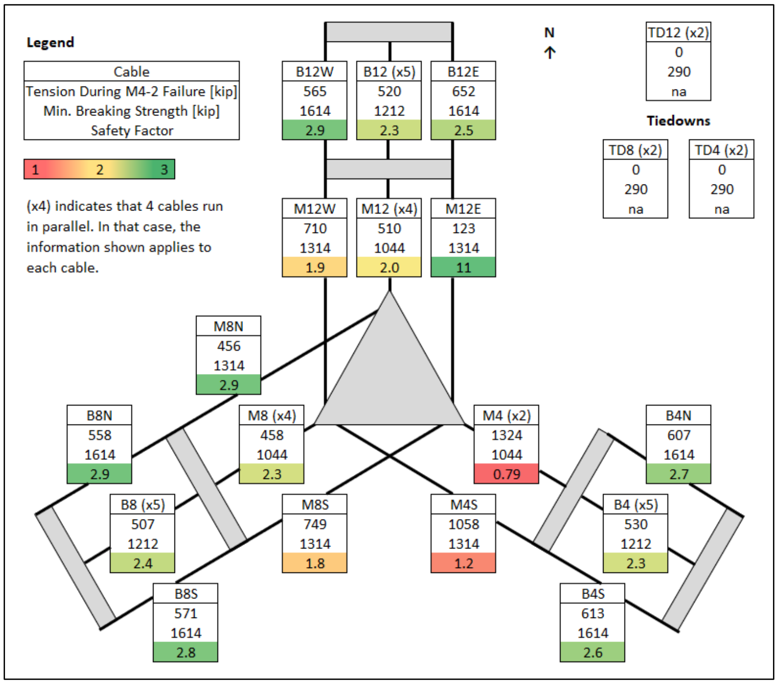

Figure 24: Hypothetical cable tensions and safety factors during M4-2 failure, with 2.0 dynamic amplification.

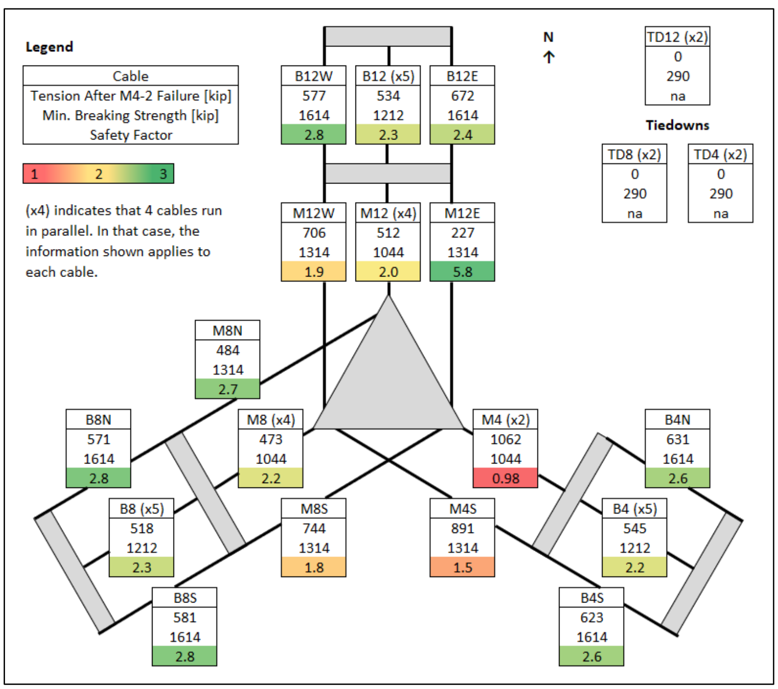

Figure 25: Hypothetical cable tensions and safety factors after M4-2 failure.